STAT 5084 # Grad Problem : SES is a special case of ARIMA

Graduate students taking STAT 5084 should complete some "grad problems" over the course of the semester. There will be around six of these problems, and you'll need to do a good job on half of them.

The goal of this problem is to show that simple exponential smoothing is a special case of ARIMA. It may help to read the ETS(A,N,N) section of fpp3 chapter 8.5 (or maybe, this exercise helps make sense of that section).

Begin with the component form of simple exponential smoothing:

$$\begin{aligned} y_t &= \ell_{t-1} + \epsilon_t \\ \ell_t &= \alpha y_t + (1-\alpha)\ell_{t-1} \end{aligned}$$

Here ϵ_t is white noise, so that the forecast equation is, as expected, $\hat{y}_t = E[y_t] = \ell_{t-1}$.

1. Show that $\ell_t = \ell_{t-1} + \alpha\epsilon_t$. This is equation (8.4) in fpp3.
2. Show that $y_t - y_{t-1} = \epsilon_t - (1-\alpha)\epsilon_{t-1}$.
3. Conclude that y is ARIMA(p, d, q). What are p, d, and q here?