STAT 5084 # Grad Problem : Fitting an MA(1) model

Graduate students taking STAT 5084 should complete some "grad problems" over the course of the semester. There will be around six of these problems, and you'll need to do a good job on half of them.

In this problem, you write code to fit an MA(1) model to data.

Suppose you are given a time series $y_1, \ldots, y_T$. An MA(1) series is described by $y_t = \epsilon_t + \theta\epsilon_{t-1}$ where ϵ is white noise with variance σ^2. Rewriting as $\epsilon_t = y_t - \theta\epsilon_{t-1}$ gives a method for computing ϵ_t inductively, assuming some value for ϵ_0.

1. Write a function that takes a time series y and a guess θ and computes the SSE for the MA(1) model with the given θ. That is, compute values $\epsilon_1, \ldots, \epsilon_T$ (assuming $\epsilon_0 = 0$), then return the value $\sum_{t=1}^{T} \epsilon_t^2$.

2. Simulate an MA(1) series with $\theta = 0.5$. Compute the SSE for evenly spaced values of $\theta \in [0.2, 0.7]$ and plot the resulting values. Is the minimum close to the actual value of 0.5?

3. Create a function that fits an MA(1) model to a given time series by choosing θ to minimize the SSE. You can use the built-in `optimize` function to numerically seek the minimum for your SSE function. Check it on a variety of simulated MA(1) series, and see that it recovers values of $\theta \in (-1, 1)$.

4. Center the built-in series `Nile` by subtracting its mean. Use your function to fit an MA(1) model to the zero-centered `Nile` series. Compare your value of θ to the one given by `arima(Nile, order = c(0,0,1))`.