STAT 5084 # Grad Problem : Mean of a random walk

Graduate students taking STAT 5084 should complete some "grad problems" over the course of the semester. There will be around six of these problems, and you'll need to do a good job on half of them.

The goal of this problem is to think about the "mean value" of a random walk model.
Let ϵ_t be white noise with standard deviation σ.
Let y_t be a random walk with $y_t = y_{t-1} + \epsilon_t$, and suppose $y_0 = \mu$.

1. Show that $E[y_t] = \mu$ for all t.
2. Suppose we want to estimate μ from $y_1, ..., y_T$. Show that $E[\bar{y}] = \mu$, so $\bar{y}$ is an unbiased estimate of μ.
3. Show that:

$$\text{Var}(\bar{y}) = \frac{\sigma^2}{T^2} \sum_{n=1}^{T} n^2 = \frac{\sigma^2}{T^2} \frac{T(T+1)(2T+1)}{6} \approx \sigma^2 \cdot \frac{T}{3}$$

4. Why does the result of part (3) mean we cannot use $\bar{y}$ to effectively estimate μ?
5. Think about this: What if we forgot about y_0 and instead started the whole analysis with y_1?